Numerical Modeling of Various Solid Propellant Motor Geometries;

Core Burning Grains, Bates Grains, and Nozzleless Motors

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My Background



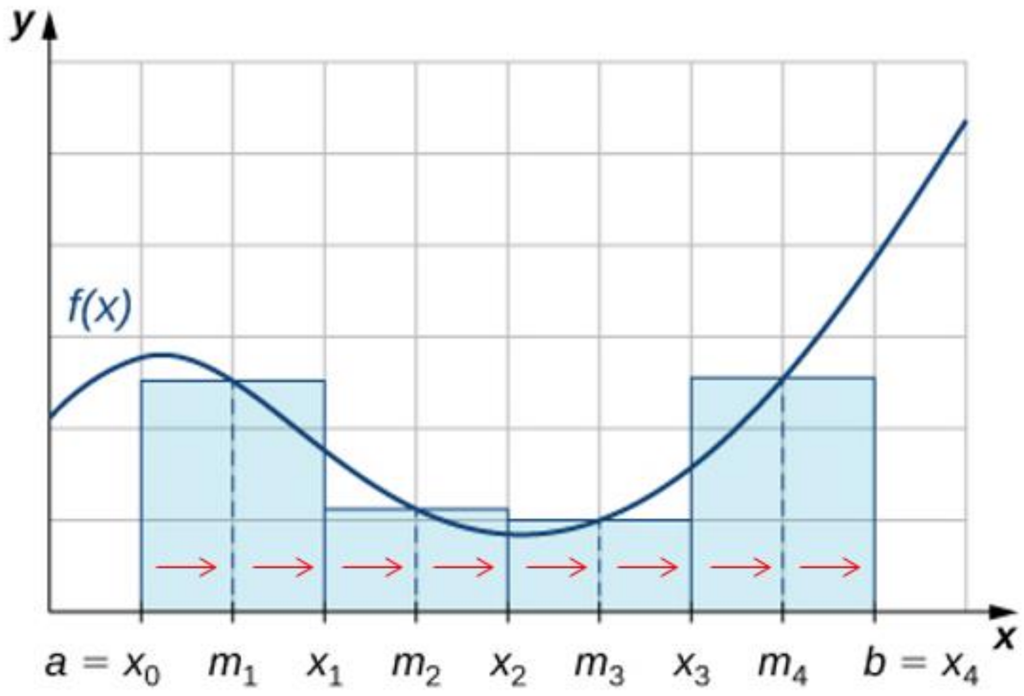


What is numerical modeling?

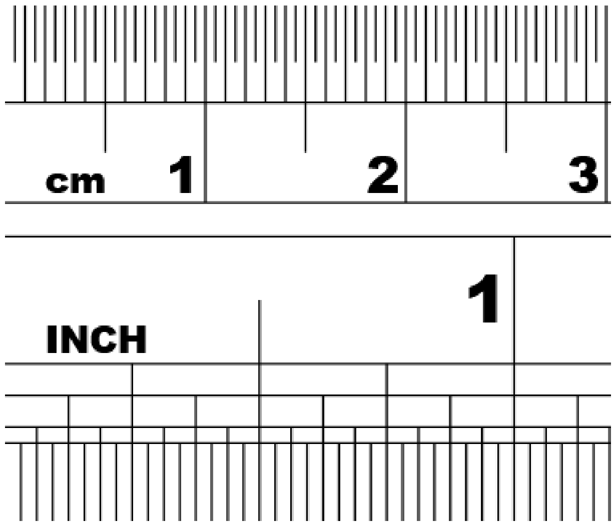
Numerical modeling, is when we take a system of equations and solve them over and over and over again, slightly adjusting the variables from one instant to the next based on the results of the previous instance.

This allows us to approximate the behavior of a complex system relative to time or some other frame of reference.

This is how motor simulation tools, such as CoreBurner work on a fundamental level.



Decisions, Decisions: Part I

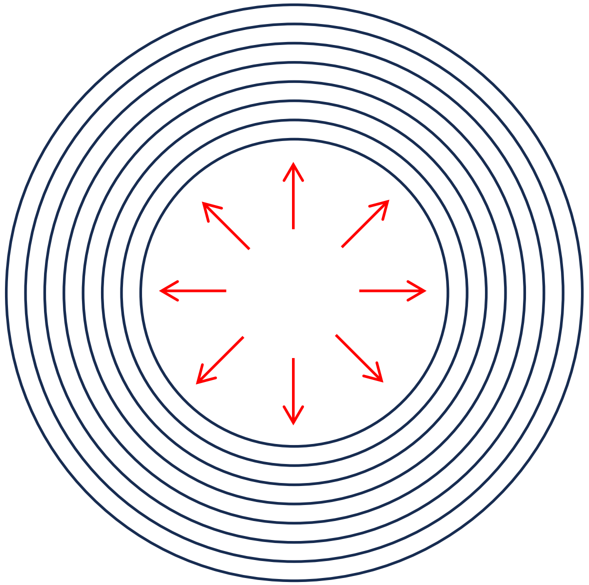


Distance?

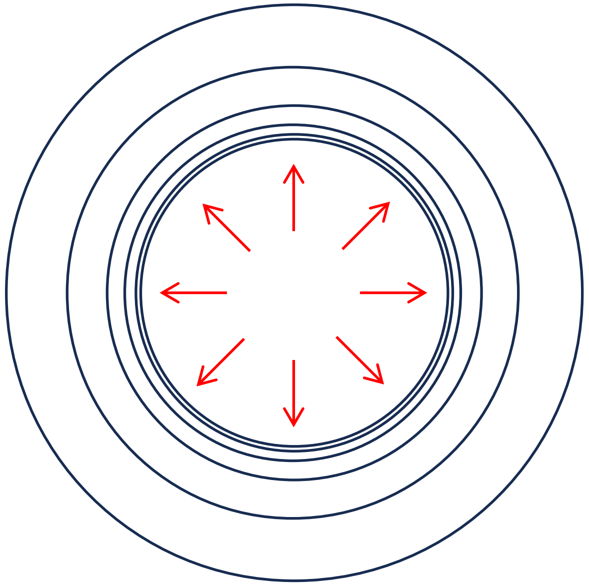
or

Time?

An Example in a Core Burning Motor w/High Pressure Exponent Propellant



Fixed Distance
Web Regression



Fixed Time
Web Regression

Decisions, Decisions: Part II

Assumptions. We all know the saying;
"Assuming makes an a$$ out of you"

We live in a beautifully complex world though and we have to make assumptions to make these problems tractable for us lowly ne're-do-wells.

And these assumptions we make materially influence the outcome of our model, so we have to decide; where do we draw the line?

Bound the Problem

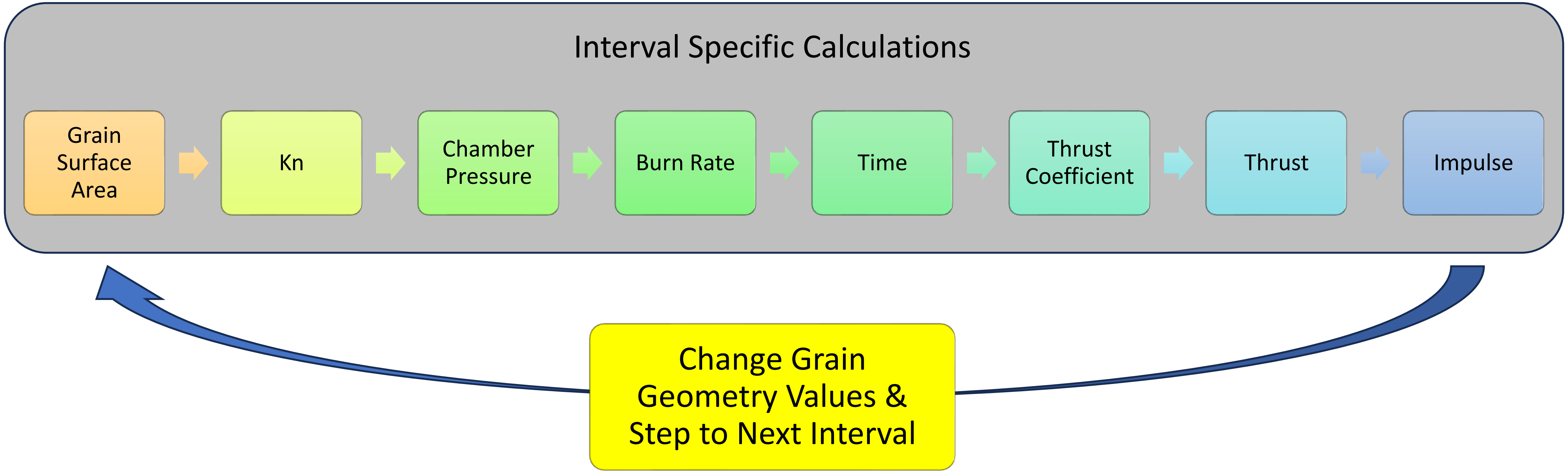
- X psi to X psi?
- Full expansion of the nozzle?
- Do we care about specifics of the nozzle geometry?
- Losses from sharp edges, turbulent flow in the nozzle, etc.?
- Do we account for erosive burning?
- What about nozzle erosion?
- Slag build up?
- Flexural modulus of the grain and it’s effect of surface area relative to chamber pressure?
- Thermal transfer from the casing or liner into the grain?
- Etc. Etc. Etc.

The Assumptions I Make

- 14.7 to 2,000 psi (this is a relatively common range that motors operate over in the hobby realm)
- Full expansion (*kind-of*)
  - My tool, CoreBurner allows the user to set the nozzle exit pressure, but defaults to 14.7 psi, therefore full expansion at sea level unless they change it
- Propellant is rigid, thermal effects other than those accounted for inherent in the collection of burn rate data are neglected
- Nozzle is 90% efficient
- Grain regression is normal to the grain surface
- No erosive burning
- Entire grain surface lights essentially instantaneously and completely
- Nozzle is fixed, no erosion, slag build up, etc.

Fundamentals of Motor Modeling

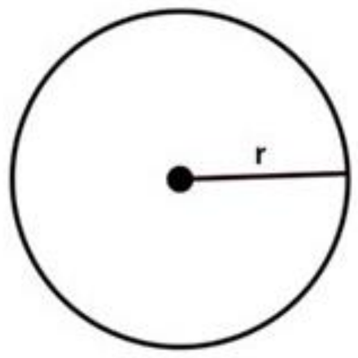
What are we calculating for each instance? What's changing throughout the burn?



Grain Surface Area (Geometry, Yikes)

Most common propellant grains can be broken up into simple geometric shapes and we can calculate the surface area of the grain that’s burning during each interval this way.

Most can actually be calculated using only the equation for area of a circle or circumference of a circle combined with the length of the cylindrical area (creating an open ended cylinder).



Area= πr^2

Circumference = $2\pi r$

Derived Equation of Surface Area, Bates Grains

$$A = (2(\pi {r_e}^2 - \pi {r_i}^2)) + (2\pi r_i l)$$

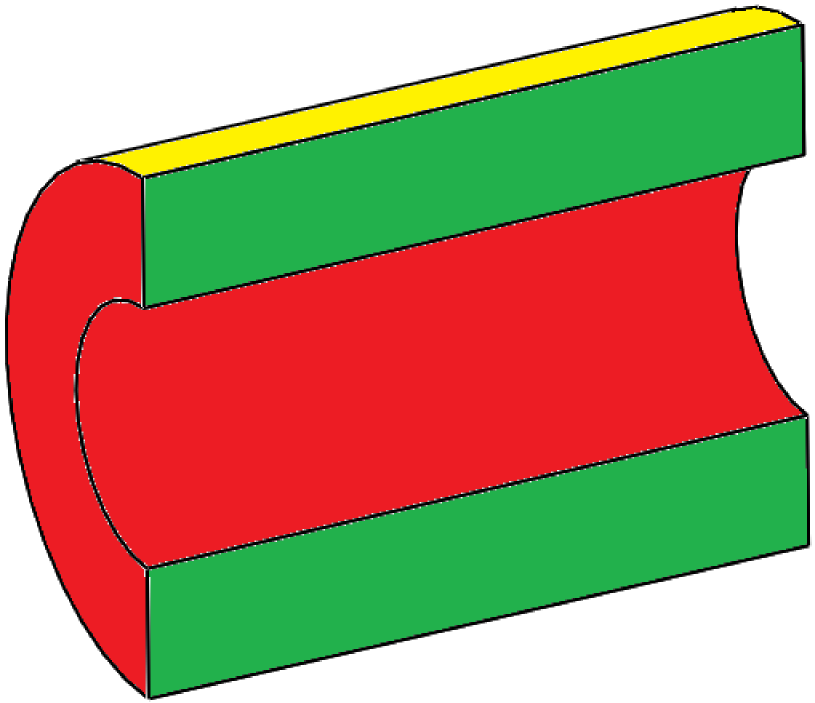
Where:

$A = Grain\ Surface\ Area\ Burning$

$r_e = Grain\ Radius\ External$

$r_i = Grain\ Radius\ Internal$

$l = Grain\ Length$



*Equation is for a single grain; if using multiple bates grains, the area is simply multiplied by the number of grains being used

Derived Equation of Surface Area, Core Burning

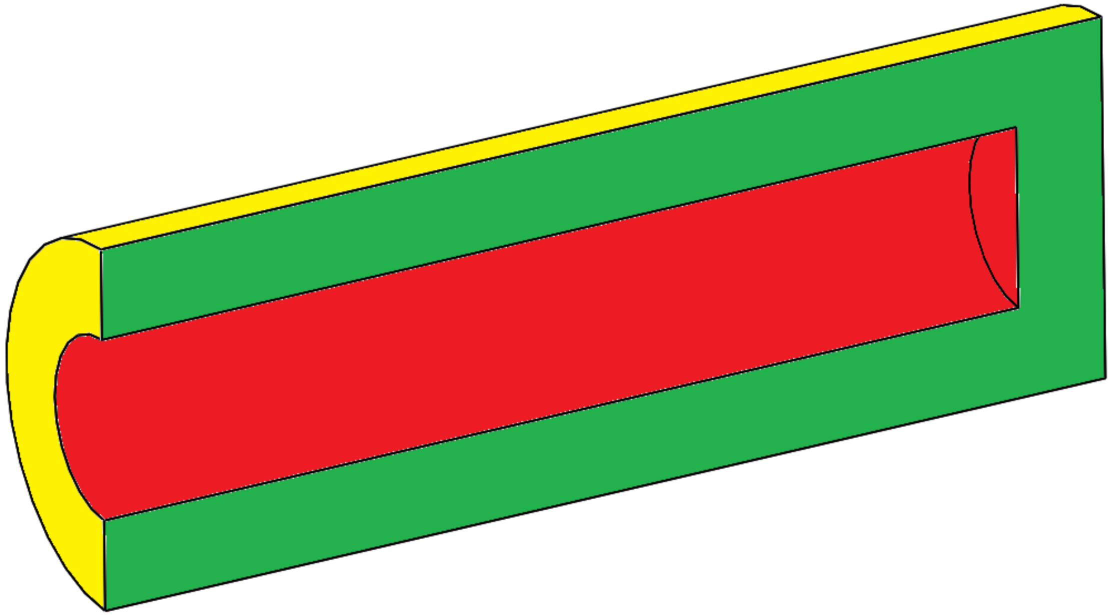
$$A = \pi r^2 + 2\pi r l$$

Where:

$A = Grain\ Surface\ Area\ Burning$

$r = Core\ Radius$

$l = Core\ Length$



*Equation assumes end of grain is inhibited; if using uninhibited grain end, the first portion of the expression uses the radius of the outside of the grain, rather than the core radius, the second portion uses the core radius

Kn (*Klemmung Number*)

The Klemmung Number or Kn for short, describes the ratio of burning propellant's surface area to the cross-sectional area of the nozzle throat.

$$Kn = \frac{Surface\ Area\ of\ Burning\ Propellant}{Nozzle\ Throat\ Area}$$

Chamber Pressure

$$P_o = (K_n a p c^*)^{\frac{1}{1-n}}$$

Where:

P_o = *Chamber Pressure*
K_n = *Klemmung Number*
a = *Burn Rate Constant*
n = *Burn Rate Exponent*
p = *Propellant Density*
c^* = *Characteristic Exhaust Velocity*

WARNING

LOOKS SIMPLE BUT UNITS FOR THIS EQUATION ARE A MESS AND MUST BE CORRECT FOR THIS TO WORK

Burn Rate & Time

$$r = aP_o{}^n$$

Where:

r = *Burn Rate*
a = *Burn Rate Constant*
n = *Burn Rate Exponent*
P_o = *Chamber Pressure*

$$t = \frac{d}{r}$$

Where:

t = *Time*
d = *Distance*
r = *Burn Rate*

Thrust Coefficient

$$C_f = \sqrt{\frac{2k^2}{k-1} * \frac{2}{k+1}^{\frac{k+1}{k-1}} * (1 - \left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}})} + \frac{(P_e - P_a)A_e}{P_o A^*}$$

Where:

C_f = *Coefficient of Thrust*
P_e = *Pressure Exit*
P_o = *Pressure Chamber*
P_a = *Pressure Atmosphere*

A_e = *Nozzle Exit Area*
A^* = *Throat Area*
k = *Ratio of Specific Heats*

Interval Thrust & Interval Impulse

$$Thrust = Chamber\ Pressure * Thrust\ Coefficient$$

$$Impulse = Thrust * Time$$

Bringing it All Together

Motor Geometry

Grain OD:	1.214	inches
Core Diameter:	0.5	inches
Grain Length:	2	inches
Nozzle Throat Diameter:	0.375	inches
Nozzle Exit Pressure:	14.7	psi
Number of Grains:	11	
Propellant Mass:	1.310	lbs

Propellant Characteristics

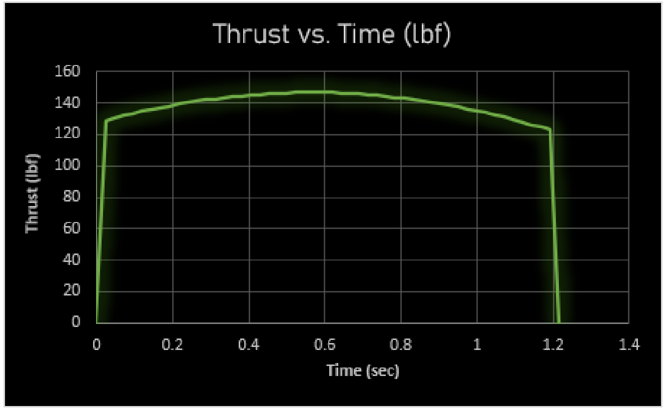
Density:	0.062	lbs/in^3
Burn Rate Coefficient:	0.0676	psi, in/sec
Burn Rate Exponent:	0.22	
C-Star:	885	m/s
Specific Heat Ratio:	1.137	

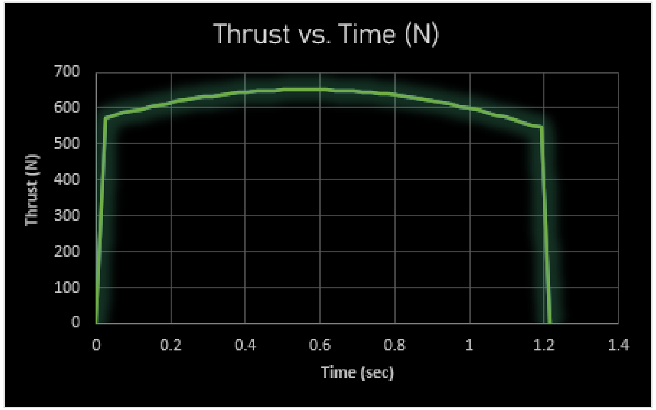
Simulation Parameters

Interval Distance:	0.01428	inches

Performance Summary

Total Impulse:	166	lb-sec
Total Impulse:	738	N-sec
Burn Time:	1.22	sec
Average Thrust:	137	lbf
Average Thrust:	606	N
Specfic Impulse:	127	sec
Max Chamber Pressure:	911	psi
Max Thrust:	147	lbf
Max Thrust:	652	N
Motor Classication:	J	

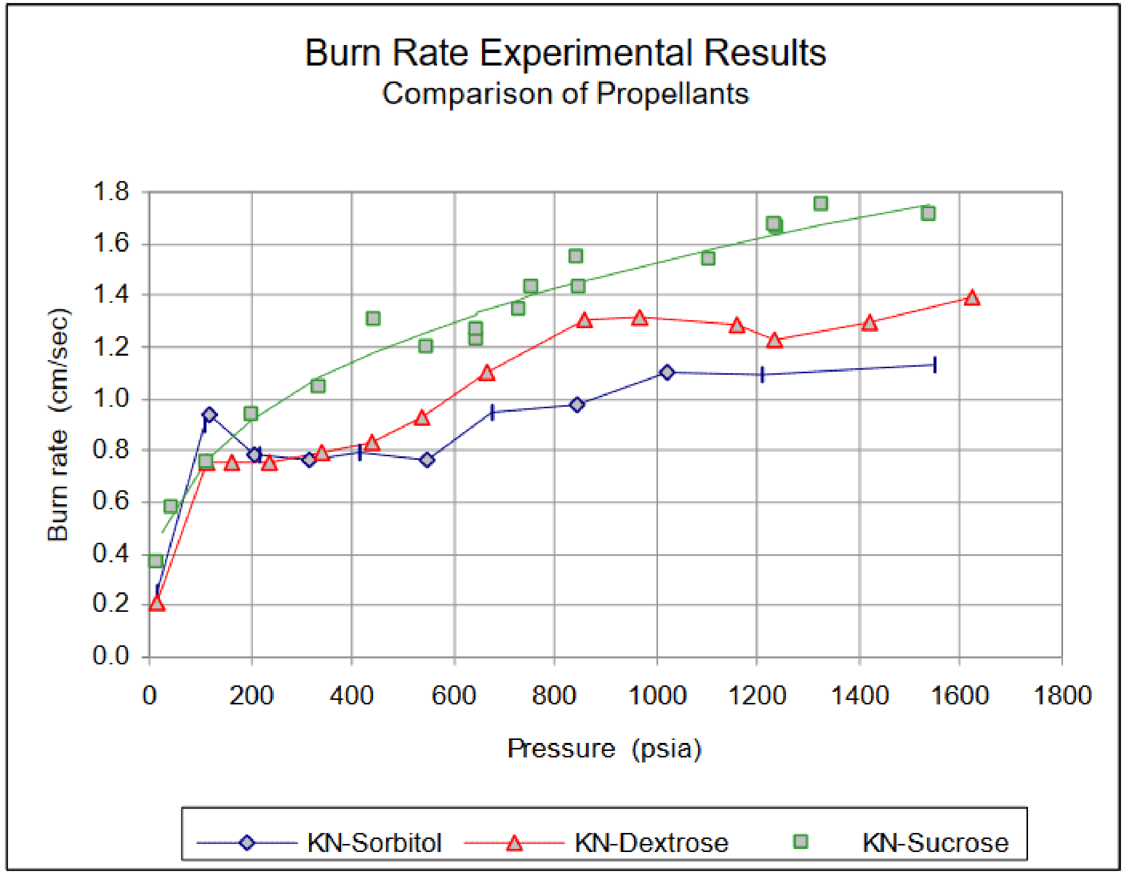




	Interval #	Core DIA (inches)	Core Length (inches)	Kn	Pressure (psi)	Thrust Coefficient	Burn Rate (in/sec)	Interval Time (sec)	Time (sec)	Thrust (lbf)	Thrust (N)	Interval Impulse (lb-sec)
	0	0.5	2	0	0	0	0	0	0	0	0	0
Ignition>	1	0.51428	2	511	810	1.44	0.295	0.024	0.024	129.006	572.786	3.123
	2	0.52856	1.986	515	818	1.44	0.296	0.024	0.048	130.515	579.488	3.152
	3	0.54284	1.971	519	827	1.45	0.296	0.024	0.072	131.961	585.907	3.180
	4	0.55712	1.957	523	834	1.45	0.297	0.024	0.096	133.342	592.040	3.206
	5	0.5714	1.943	527	842	1.45	0.298	0.024	0.120	134.659	597.884	3.232
	6	0.58568	1.929	530	849	1.45	0.298	0.024	0.144	135.909	603.438	3.256
	7	0.59996	1.914	534	856	1.45	0.299	0.024	0.168	137.094	608.698	3.278
	8	0.61424	1.900	537	862	1.45	0.299	0.024	0.192	138.213	613.664	3.300

A Bit of Nuance; Plateau or Mesa Propellants

That valid for 14.7 to 2000psi assumption I made earlier is bullshit. *For some propellants.*



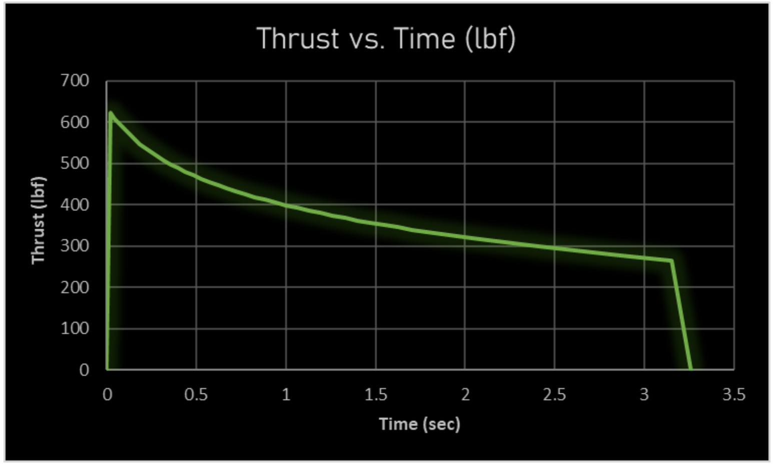
Graph/Propellant Data Courtesy of Richard Nakka

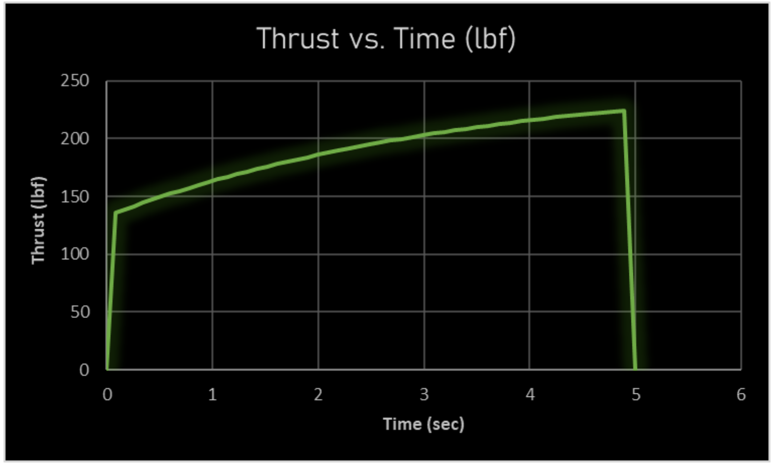
A Bit More Nuance; Nozzleless Motors

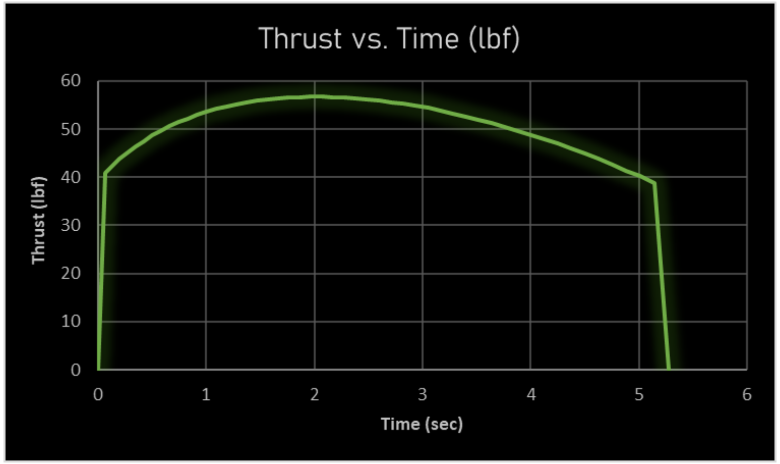
Nozzleless motors use the core of the grain itself as the functional "nozzle throat".

- Thrust coefficient for the nozzle is just "1" (unless you have a divergent section molded into your grain, but we're not going there today).
- Nozzle throat cross-sectional area changes with each interval with the grain geometry
  - Pressure is generally highest at ignition
  - Depending on the propellant selection and length of the grain these motors can produce progressive, regressive, or more neutral thrust profiles

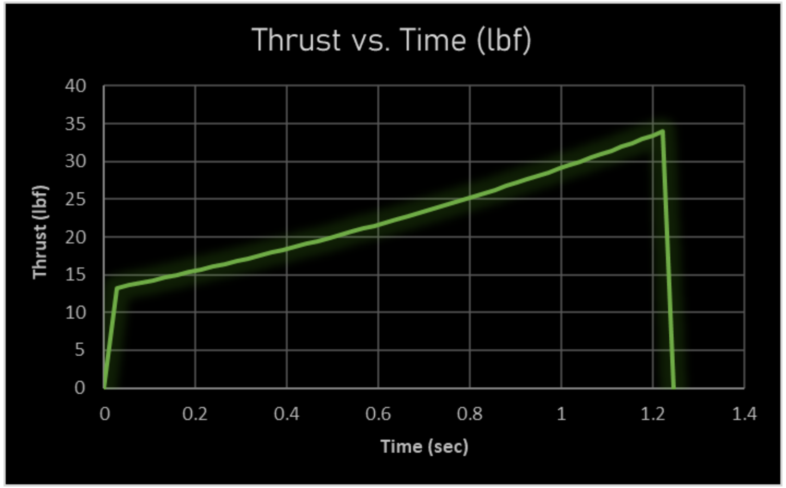
Example of a "Real" Nozzleless Rocket: Courtesy of L3 Harris

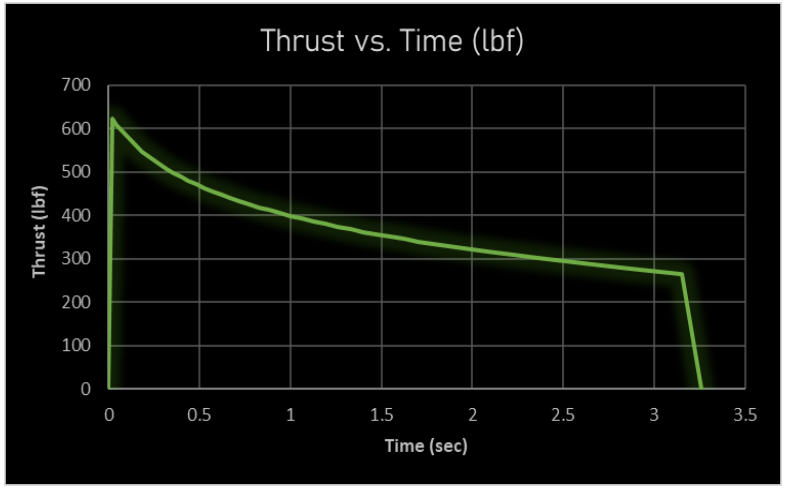


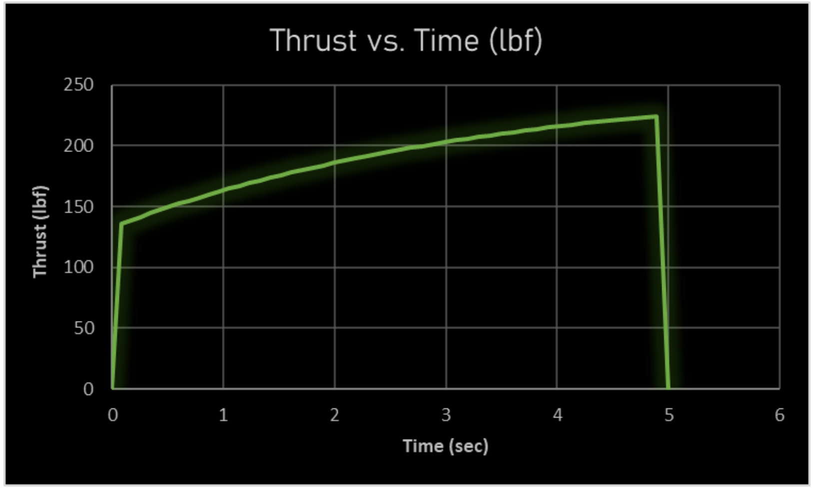


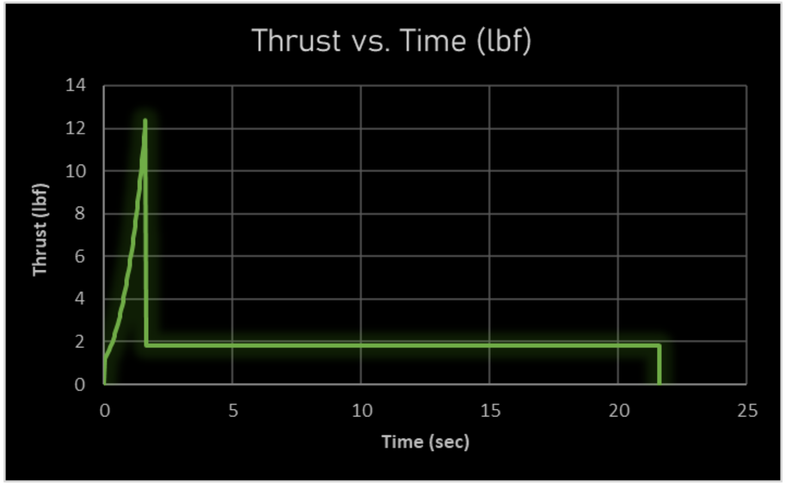


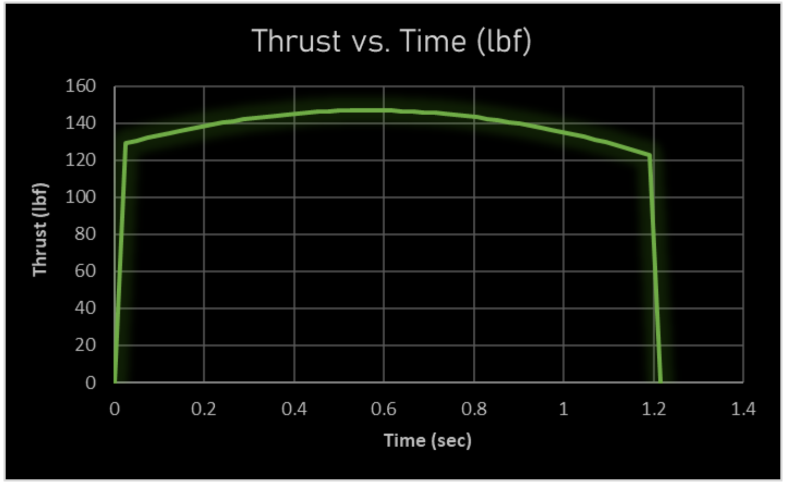
The Possibilities Are Endless

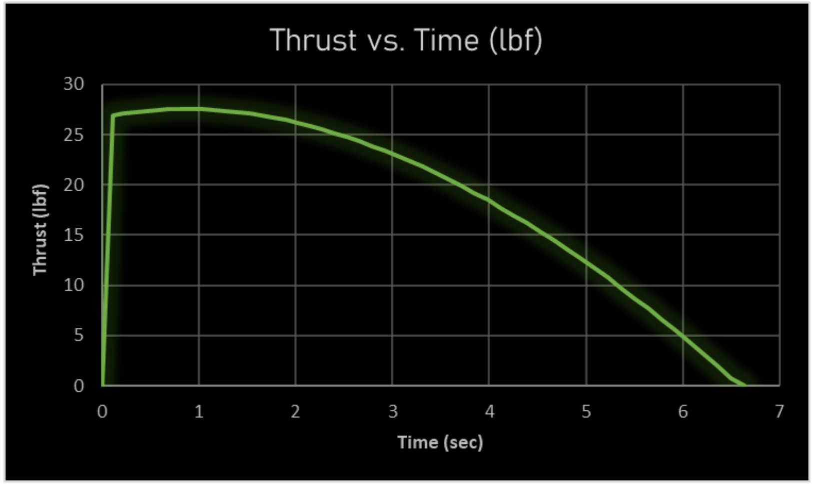




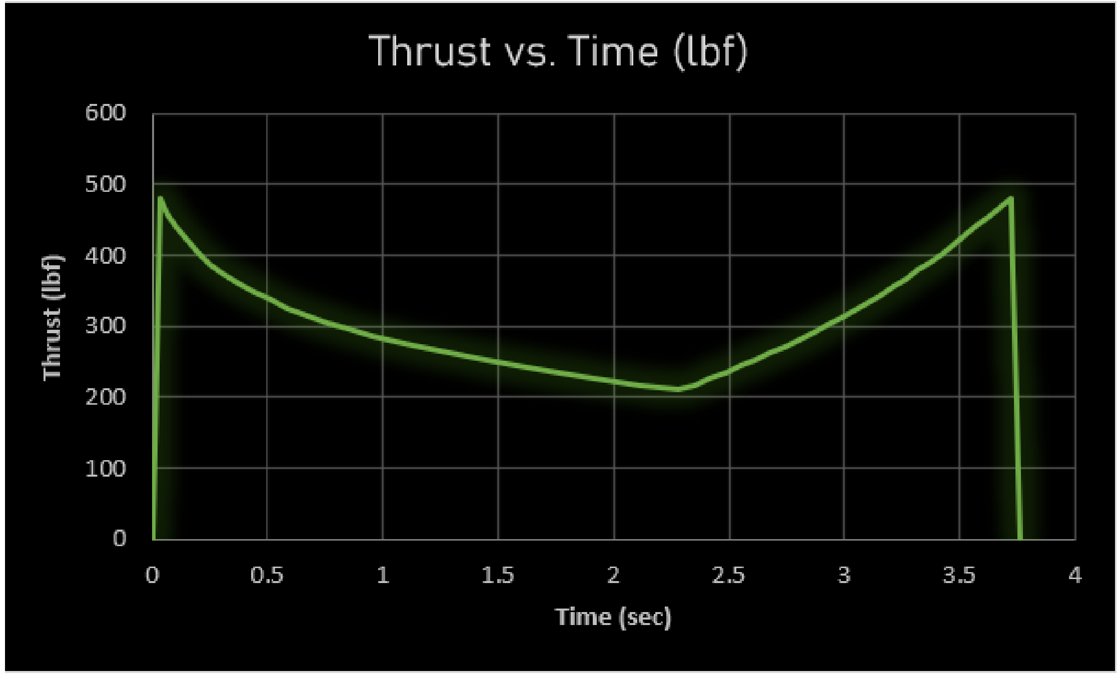








A Novel Grain Geometry Example



Predicted Thrust Curve Profile

"Two-Phase" Nozzleless-to-Embedded Nozzle Motor

Questions?